ABSTRACT: A comprehensive testing database composed of modeling-quality, multi-directional cyclic simple shear testing on medium to high relative density, fully-saturated samples of Monterey 0/30 sand has recently been developed. This testing incorporated a variety of never before examined multi-directional stress paths on both level and sloping ground. Results from these tests have proven useful for enhancing current understanding of liquefaction behavior by allowing for a more complete theory to emerge. This new three-dimensional theory greatly expands current understanding of liquefaction behavior and elucidates some areas in which current theory—which has been based principally on uni-directional testing—can be misleading or unconservative. Of particular interest are the topics of (a) pore pressure generation as a function of loading magnitude and direction, (b) the effects of sloping ground conditions, (c) the post-liquefaction pore pressure behavior within each cycle, and (d) strain accumulation. Also described is the impact of multidirectional loading on maximum achievable pore pressures.

INTRODUCTION

In recent years, significant research effort has been focused on developing state-of-the-art constitutive soil models capable of accurately predicting the cyclic and permanent deformations of liquefiable materials over the small to moderate strain range (<1m). Unfortunately, the use of advanced constitutive models for liquefiable soils is being hampered by a lack of the high-quality laboratory testing for use in the validation of existing models and development of new ones. In particular, very little
modeling-quality testing has been performed on liquefiable materials experiencing multi-directional stress (or strain) paths, medium dense to dense sands that exhibit dilative behavior, and materials under initial “driving” shear conditions as would be found in under sloping ground or under a structure.

To address this need for modeling-quality data a program of multi-directional simple shear testing was performed on medium to high relative density, fully saturated samples of Monterey 0/30 sand. A variety of previously unexplored multi-directional stress paths were imposed. These stress paths can be separated into three general categories: linear, oval/circular, and figure-8, as shown schematically in Figure 1. Tests were performed both with and without an initial static driving shear stress in order to replicate the in-situ loading conditions on soils elements under both sloping and level ground conditions. Additionally, the major axis of the loading types was oriented in both the dip and strike directions for the sloping ground cases. A second related series composed of uni-directional tests was performed in conjunction with the multi-directional program.

The goals of these testing series are twofold. The first is the development of a high quality laboratory test database describing the behavior of liquefiable soils for use in model development and calibration. The second is the development of insight into the behavior of liquefiable soils under generalized loading conditions. The attainment of both of these goals required specialized sample preparation and testing techniques to be developed such that both the soil fabric and the imposed loading replicated the in-situ conditions as much as possible.

This paper does not present an exhaustive discussion of the lessons that can be learned from this testing series or of the testing techniques employed. Instead, the focus has been placed on topics of particular interest in model development. For a more complete discussion of the results of this testing program, the reader is directed to Kammerer et al. (2002) and Wu et al (2003).
OVERVIEW OF TESTING PROGRAM

A comprehensive uni-directional test series was performed simultaneously and in conjunction with the bi-directional testing series described here in order to develop a well-verified basis from which to assess the influence of a load in a second horizontal direction (Wu et al., 2003). This testing incorporated four relative densities (30, 45, 60 and 80%), and three initial vertical effective stresses (40, 80 and 1.5 kPa). The liquefaction susceptibility curves produced by this baseline series for all densities and initial vertical effective stresses are shown in Figure 2. In this testing program, triggering was defined as 3% single amplitude or 6% double amplitude shear strain. The curves for 80kPa initial vertical effective stress can be used to compare the bi-directional series, which also incorporated a confining stress of 80kPa. Post-liquefaction settlements and strains were also recorded for the uni-directional testing series.

The bi-directional series performed is shown schematically in Figure 1. Testing was performed at 80kPa and on medium density and high density samples. Because no additional methods were used to further densify the samples after they were built and the top cap was installed on the sample, the densities show some scatter. A range of CSR values were used. Because different stress paths had very different resistance to liquefaction, CSR values were chosen on a case-by-case basis, but the values were typically near 0.11, 0.22 or 0.44.

![FIG 2: Liquefaction triggering curves for Monterey sand under level ground conditions (Wu et al, 2003)](image-url)
LABORATORY TECHNIQUES, EQUIPMENT & MATERIAL

Testing was performed in the U.C. Berkeley Bi-directional Simple Shear Device (Boulanger et al., 1993) which is capable of applying horizontal shear stresses in two orthogonal directions due to the dual track and piston system. Cell and back pressures can also be applied, which allows for testing of fully saturated samples. It is important to note that when developing constitutive models, only simple shear and torsional shear data should be used. Triaxial testing is not considered appropriate for this purpose as the instantaneous 90 degree change in the orientation of the principal stress axes is substantially different from in situ seismic loading in which a smooth rotation of the principal stress stresses occurs.

For this testing series fully-saturated, clean sand samples were constructed using wet-pluviation. The samples were contained in NGI-style wire-reinforced latex membranes that hold the sample diameter constant to assure that $K_o$ conditions are maintained. While this method does a good job of assuring $K_o$ conditions throughout the test, the actual lateral pressures applied to the sample throughout testing are unknown. For the cases where an initial horizontal static shear stress was applied to replicate sloping ground conditions (i.e. $\alpha$>0), the horizontal stress was applied incrementally in proportion to the vertical load applied at each time step. This assured that the sample was not artificially sheared during sample preparation.

Load control was used for both the horizontal shear stresses and the vertical normal stress. This differs from the most common testing method (typically performed on dry samples) in which a constant height is maintained and the vertical load is allowed to drop. The benefit of this new method is that it allows for a direct measure of pore pressure to be recorded during testing. In the traditional method, the drop in vertical load is equated to the pore pressure that would occur in a fully saturated sample. The traditional method is adequate for most purposes because it is often only the number of cycles to liquefaction are desired. However for modeling purposes, the behavior at each point in the test should be as close to the true in-situ behavior as possible, thus the new testing method was used.

Testing was performed on Monterey #0/30 sand, a highly uniform, medium sized, sub-rounded beach sand that was further prepared by the washing of the material over a #200 sieve to remove fine material. The $D_{50}$ for the material is 0.36mm, the coefficient of uniformity ($C_u$) is 1.29, and the coefficient of curvature ($C_c$), is 0.98. A minimum void ratio of 0.541 was determined by the modified Japanese method and a maximum void ratio of 0.885 was determined by the dry tipping method. Additional information on the laboratory program and material is provided in Kammerer et al (2003) and Wu et al. (2003).

PARAMETERS, TESTING DEFINITIONS, AND FIGURES

Parameters and Testing Definitions
Numerous terms and parameters have traditionally been used to describe both the state of a sample and the loading applied. However, some parameters require further specification to remove ambiguity arising from the multi-directional loading and others must be newly developed. When developing new parameters and clarifying
existing ones, care has been taken to assure that they can be extended to irregular loading without confusion. The definitions proposed here provide a common and effective set of parameters that will be increasingly needed as multi-dimensional models are developed and compared with the new data available.

Several commonly used terms have been made more precise by the addition of subscripts such as “dip”, “strike”, “X”, “Y”, “max”, and “min”. The first two subscripts relate to sloping ground tests and denote the value in the direction of the initial static shear stress (or slope) and perpendicular to the initial stress, respectively, as shown in Figure 3. The geology-based terms dip and strike are used because they have precise unambiguous definitions related to sloping-ground conditions in the field. If no slope exists the terms “X” and “Y” can be used to differentiate the horizontal directions. The terms “max” and “min” relate to a wide variety of parameters. They can connote the maximum direction (e.g. $\text{CSR}_{\text{max}}$) or the maximum absolute values within a given cycle or tests (e.g. $\gamma_{\text{max}}$).

If common terms (e.g. shear strain ($\gamma$) or shear stress ($\tau$)) are used without subscripts, the magnitude of the parameter—regardless of direction—is intended. For example $\gamma_{\text{dip}}$ denotes the strain in the direction parallel to the initial static shear stress, while $\gamma$ denotes the largest magnitude of shear strain, regardless of the direction of the strain.

It is important to note that it is the magnitude of normalized shear stress and shear strains that are presented in many of the figures provided in this paper. This is done because (1) the choice of “principal” direction is often arbitrary and (2) the data is far more meaningful when the magnitude is used. This point is stressed because the plots are similar to those the reader is likely used to and this change can potentially lead to misunderstanding, as discussed in more detail below.

**FIG 3: Multi-Directional Parameter Definitions**
Among the common definitions used in this paper are the \( r_u \), \( \alpha \), and CSR. \( r_u \) is the normalized pore pressure as commonly defined. The \( \alpha \)-value is the normalized initial static shear stress to which the sample has been consolidated \((\tau_c/\sigma'_c)\). A non-zero \( \alpha \)-value implies that the test is replicating either the presence of a structure or sloping ground conditions during testing. This initial shear is maintained throughout testing and any cyclic load applied is, therefore, offset by this value. The \( \alpha \)-value is essential an initial driving SSR, as shown in Figure 3.

CSR has been defined as the change in normalized shear stress from the central value of the cycle. In multi-directional testing the CSR must be defined in a specified direction. Values must be clarified with a subscript as both \( \text{CSR}_{\text{max}} \) and \( \text{CSR}_{\text{dip}} \) are of interest in sloping-ground testing.

The new term, shear stress ratio (SSR), has been used in this paper to note the normalized shear stress magnitude \((\tau/\sigma'_c)\) at a particular point in time. This should not be confused with CSR. Unlike CSR, SSR is defined at each time-step during loading (and is continually changing). Another new term is the normalized effective vertical stress. This is defined as \((\sigma'_{v}/\sigma'_c)\) and (like SSR) is defined at each point in time. This parameter is useful because (1) it allows for direct comparison of vertical effective stress and \( r_u \) at any point in a test \((\sigma'_{v}/\sigma'_c\approx 1-r_u)\) and (2) tests with different initial vertical effective stresses can be directly compared.

**Multi-Dimensional Plots**

There are three general categories of plots that have been developed to present the results from the bi-directional testing presented here. These are the “standard 4-way” plots, the plan view stress and strain plots, and the plan view effective stress

![FIG. 4: Standard 4-way plot for Test#Ms46cyck (CSR\(_{\text{max}}\)=0.23 \(\alpha\)=0.11)](image-url)
plots. These plots are designed to present most of the basic information currently provided for uni-directional tests, while still accommodating multi-directional loading. An example of the 4-way plot is shown in Figure 4. This series of 4 graphs are organized such that each two adjacent sets of plots have the same axes. (Note that the normalized effective vertical stress is \( \gamma \). Thus these two axes are simply inverted.) As a result, only four independent data series are presented. These data series include the shear stress ratio (SSR) the normalized pore pressure ratio (\( r_u \)) the number of cycles, and the strain magnitude.

The first quadrant of the sample 4-way plot shows the magnitude of SSR (regardless of direction) and the normalized effective vertical stress. This is somewhat analogous to stress-path space commonly used in modeling. The data is presented in this form because the applied lateral stresses are unknown. However, the failure envelope is still evident in these types of plots. The second quadrant is the stress-strain plot. Admittedly, it is typically difficult to read for the bi-directional tests. The third and fourth quadrants show the normalized pore pressure (\( r_u \)) and the shear strain (\( \gamma \)) as test progresses. Because strain magnitude is always positive and non-directional, this plot can not be used to determine the (single or double amplitude) strain range achieved during a given cycle.

Care should be taken when viewing the 4-way and other “standard” plots as misunderstandings can result. For example, because the strain plots often reduces deformations in two dimensions into a single value, information is lost, as shown schematically in Figure 5. The top axis shows the horizontal stresses in plan view for two different bi-directional stress paths. The middle axis shows typical (though idealized) horizontal strains which would results (also in plan view). The lowest axis shows how these strains would look when presented on a standard plot showing \( \gamma \) versus the number of cycles or time. Clearly, the strain information provided on the standard 4-way plots alone is insufficient to fully describe the actual strains the sample undergoes, yet the information is very useful and necessary. To prevent misunderstanding plan view strain plots should be provided with the standard strain plot shown in Figure 4. It is recommended that the reader compare Figures 4 and 6. The second set of plots used to present the data provides the shear stresses (as SSR) and strains in plan view, as shown in Figure 6. To allow for rapid visual assessment,
the two orthogonal axes are of equal scale in each plot. The degree of both stress rotation and stress release is readily apparent in these plots. The relationship between the general stress path category and the initial static shear stress is also easily discerned by noting the location of the zero axis as compared to the center of loading. Examination of the plan view plots provides insight into the relation between stresses and strains in two-directional loading.

The final type of plot provided is the plan view effective stress plot, which presents the vertical effective stress throughout a single post-liquefaction cycle. An example of this plot is provided in Figure 7 for the same test shown in Figures 5 and 6. The diameter of the bubble represents the value of the normalized effective vertical stress (on the axes scale shown) and the center of the bubble is located at the horizontal shear stresses corresponding to that point. The cycle shown for each of the tests is the one that occurs immediately after the limiting pore pressure, $\tau_{u,\text{lim}}$, has been reached.

**RESULTS**

When considering the differences in behavior between uni-directional and multi-directional loading, it is useful to first recognize that there are two distinct phases in each test. The first is the period of excess pore pressure generation and accumulation, which occurs from the start of the test to the point where the failure
envelope is met. During this period, the maximum pore pressure for each cycle increases until an asymptotic value is reached—at which time it becomes relatively constant during subsequent cycles. The maximum asymptotic value of the normalized pore pressure is the limiting pore pressure \( (\hat{r}_{u,\text{lim}}) \). An example of this behavior can be seen in Figure 8 (D, F, and G). Because \( \hat{r}_u \) at each point is equivalent to \( 1-(\sigma_v'/\sigma_C') \), the x-axes of these three plots are equivalent. The second phase occurs once \( \hat{r}_{u,\text{lim}} \) has been reached and the pore pressure behavior within each cycle repeats itself.

Researchers developing constitutive models must be able to accurately replicate each of these two phases. The pore pressure development phase may provide the bigger challenge of the two as the behavior that leads to pore pressure development under multi-directional loading appears from the data to be difficult to accurately predict. By contrast, the data presented here has readily provided insights into the relationship between load path, pore pressure, and strain in the second phase of the test, though further research and refinement is required.

**Multi-directional Loading and Triggering**

Not surprisingly, results from this study indicate \( \hat{r}_{u,\text{lim}} \) values are typically reached more rapidly under multi-directional loading than in “equivalent” uni-directional tests (i.e. tests with the same \( D_i \) and \( \text{CSR}_{\text{max}} \)). The rotation of stresses in the horizontal plane—which occurs only in multi-directional loading—likely plays a

**FIG 8: Comparison between linear and oval tests**
large role in the increased rate of pore pressure development by aiding particle rearrangement and densification of the material.

When examining results, useful comparisons can often be made with similar research. Only three previous simple shear testing programs have been conducted using multi-directional load paths (Boulanger and Seed (1995), Ishihara and Yamazaki (1980), and Ishihara and Nagase (1988)). Of these, two used regular load paths and one used real time histories. The regular load paths applied in previous series are shown schematically in Figure 1.

The results presented here are consistent with findings from Boulanger et al. (1991, 1995) and Ishihara and Yamazaki (1980), who conducted a program in which oval and circular stress paths were imposed on Toyoura sand samples.

Figure 9 presents results from both the new testing program and the previous study. The Ishihara and Yamazaki data are presented in the upper plot, which shows the number of cycles to failure for level ground tests with a variety of aperture ratios (AR), as it is defined in Figure 9. An AR of 0 is a uni-directional test, an AR of 1 denotes a circular load path, and intermediate values denote ovals.
The lower plot presents results from the new testing series. The curve for AR=0 is estimated from the triggering shown in Figure 2 for a relative density of 65%. Also shown are results from individual oval and figure-8 tests (AR=0.5) and circular tests (AR=1).

The results from these studies show similar trends in behavior, even though the sands, relative densities, and initial confining stresses used in the two studies differ significantly. These trends include a general decrease in liquefaction resistance with both an increased aperture ratio (AR) and increased cyclic stress ratio (CSR<sub>max</sub>). Both studies also show a similar reduction in additional effect as the aperture ratio increases.

While the behavior of the samples tested under typical CSR values are consistent, the behavior of some tests (42, 44, and 68) is somewhat unexpected. Unlike in the study by Ishihara and Yamazaki (1980) this research found that in some cases the application of a second direction of loading could be detrimental. These are tests with very high CSR<sub>max</sub> values that are expected to fail within a cycle or two. The reason for this seemingly counter-intuitive behavior is discussed in more detail in the following sections. While the effect of a second horizontal loading direction in these oval/circular tests seems relatively well behaved, the impact across loading categories—particularly those incorporating sloping ground—is harder to simplify and model. Research to develop quantitative rules for pore pressure generation under multi-directional loading is ongoing.

**Effect of Initial Static Shear Stress on Stress Reversal and Pore Pressure Generation**

Liquefaction cannot occur unless individual soil grains can move relative to each other and densify. This densification occurs as a result of the rapid application of shear stresses on the soil. Most sand layers in the field require cyclic loading to densify the material sufficiently to liquefy it. For this reason, shear stress reversal has long been known to play an important role in liquefaction.

The influence of stress reversal is relatively straightforward in one-directional testing. However, even the basic definition of “stress reversal” must be reassessed when considering multi-directional loads. In uni-directional testing, the term “stress reversal” implies that stresses are completely removed and then applied in the opposite direction twice in every cycle. Additionally, the incremental change in stress always lies within a single line. This can be seen in the uni-directional test shown in Figure 8.

By contrast, in multi-directional loading the shear stresses can fully change direction (i.e. rotate 180 degrees) without going through a point of zero shear stress, as can be seen in Figures 6 and 7. Clearly, in multi-directional testing stress reversal can imply either (1) stress removal/release (as in a uni-directional test), (2) rotation of the stress on the horizontal plane, or (3) some combination of both. In multi-directional loading the terms “stress release”, “stress release and reversal”, and “stress rotation” may be more appropriate. In actual seismic loading, the motion almost universally includes some combination of stress release and rotation.

In uni-directional testing, a sufficiently large initial static shear stress ratio (i.e. high α) can decrease the potential for excess pore pressure generation by keeping the
grains locked together. This occurs where $\alpha$ exceeds $\text{CSR}_{\text{dip}}$. Interestingly, the unlocking of soil grains in uni-directional loading requires not only the removal of shear stresses, but also some loading in the uphill direction. In the past, uni-directional simple shear tests with very small amounts of stress reversal have shown the same limited behavior as tests with no stress reversal (Boulanger & Seed (1995), Harder & Boulanger (1997), Vaid & Finn (1979)). Very high $\alpha$-values can also prevent liquefaction in multi-directional tests, though the $\alpha$-values required may be much higher than the equivalent uni-directional tests, depending on the stress path. Evidence of this behavior can be seen in Figure 10 which compares two tests, with similar CSR$_{\text{max}}$ and relative density values, but with increasing $\alpha$-values. In contrast to uni-directional loading, there are also stress paths for which, a very small amount of reversal can produce large pore pressures. This results in an abrupt change (and some volatility) in behavior.

The higher $\alpha$-value in test (B) of Figure 10 resulted in limited stress rotation, but no stress reversal in the dip direction. The resulting changes in both pore pressure generation (shown as maximum and minimum values for each cycle) and strain behavior are striking. The push in the uphill direction in test (A) allows the principal horizontal stresses to rotate a full 360 degrees. In test (B) there is a rotation of stresses of approximately 90 degrees. This can be further contrasted with uni-directional testing in which 180 and 0 degrees of reversal would occur for these CSR$_{\text{max}}$ and $\alpha$-values, respectively.

Figure 11 further shows the importance of stress rotation in multi-directional tests by comparing tests with similar CSR$_{\text{max}}$ values and degrees of stress reversal in the dip (downhill) direction, but with different orientations. There are two tests with different densities shown for each orientation. In this series one set of tests has a much higher degree of rotation, while the other has a much higher maximum shear

**FIG 10: Comparison of oval dip-oriented tests with increasing $\alpha$-values**
stress. The results were consistent between both sets of tests, with the strike-oriented tests showing both larger cyclic and permanent strains. This indicates that for this set of test conditions, the amount of shear stress reversal/rotation is more important than the maximum shear stress imposed. It may be noted that the plots of shear strains of tests the strike-oriented tests show permanent movement in both the strike and dip directions, though the loading seems symmetric. This is because at each point when the highest pore pressure (and highest degree of softening) is reached, the load path is moving in the same counter-clockwise direction.

**Relationship between Shear Stress and Post-Triggering Pore Pressure Behavior**

Once the limiting pore pressure ($r_{u,\text{lim}}$) has been achieved in a given test, the pore pressures measured throughout each cycle of uniform loading show repetition of behavior. Fig 12 shows the limiting maximum and limiting minimum pore pressures ($r_{u,\text{lim}}$ and $r_{u,\text{lim,min}}$) for each liquefied test in the series plotted against the shear stress ratios at which each of these values occurs. The parameter $r_{u,\text{lim,min}}$ is defined as the minimum pore pressure that occurs in a test with regular loading after $r_{u,\text{lim}}$ has been achieved. As an example, the $r_{u,\text{lim}}$ and $r_{u,\text{lim,min}}$ values plotted for tests MS36cyck are highlighted by the circles on Figure 8(G). This limiting pore pressure data exhibits a roughly inverse-linear relationship with shear stress. The few values that plot noticeably below the trend lines are cases where there was no shear stress reversal in the dip direction, but where the sample still liquefied due to large shear stresses in the strike direction (e.g. Tests MS60cyck and MS69cyck in Figure 11). In addition, samples that do not soften would plot at much lower pore pressures than would be expected from this relationship.

**FIG 11: Comparison of oval tests with no reversal**
These results are consistent with those obtained in an earlier program for an equivalent parameter by Boulanger and Seed (1995). This earlier testing series on Sacramento River Sand used a linear stress path that included an initial static driving shear stress (i.e. $\alpha > 0$) and was oriented in the strike direction, as shown in Figure 1. The lines from this earlier program (as shown on Figure 12), bound the plots of $r_{u,\text{lim}}$ versus $\alpha$ for the tests performed. Because $\alpha$ was equivalent to $SSR_{\text{min}}$ in this series, a direct comparison can be made.

This inverse relationship between shear stress and post-triggering pore pressure can be explained by dilation of the soil particles. In both uni- and bi-directional tests, dilation causes a drop in pore pressure as limiting strains are reached and particles are forced to move over each other. In the case of uni-directional loading, this relative motion must occur within a single plane. In multi-directional loading there is more freedom of movement, making the resulting dilation less problematic but still evident.

There are also cases, however, when the drop in pore pressure due to shear stress is large enough that the soil cannot soften or softens more slowly than expected. This occurs with certain load paths in which the shear stresses imposed remain very large. This is the case with Tests MS42cyck, MS44cyck and MS68cyck in Figure 9.

The inverse relationship between SSR and $r_u$ is foreseeable because the values plotted are principally located on the failure envelope for each test. For example, in Figure 8(G) every point of the final cycle remains on the failure envelope. It is only during unloading in some stress paths that the stress state of the sample does not plot

![Graph showing normalized pore pressure values achieved during a "liquefied" cycle as a function of the shear stress ratio.](image)

FIG 12: Normalized pore pressure values achieved during a “liquefied” cycle as a function of the shear stress ratio

on the failure plane.
Shear Stress Rotation and Pore Pressure Lag

As shown in Figure 8(F), there is a lag in pore pressure during the unloading phase of a uni-directional test. While some multi-directional tests exhibit this behavior, many others do not. Results from this study indicate that this lag in pore pressure is eliminated if sufficient rotation of the shear occurs during the unloading phase of a cycle. For example, stress path categories in which a high degree of rotation constantly occurs (e.g. the oval tests in Figures 4 and 8) show no pore pressure lag at all.

Many figure-8 tests represent a mix of the two conditions with both a high degree of rotation and a high degree of release occurring in different part of the stress path. Fig 13 presents a single cycle of a figure-8 tests that has each been separated into four regions. In regions A and C shear stresses are being removed then reapplied with little stress rotation. In regions B and D loads change to a lesser extent, while a higher degree of rotation is occurring. It can be seen that a pore pressure lag occurs in this test, but only where little shear stress rotation in the horizontal plane occurs during unloading (parts A and C). Once the material rotates (vertically) through the point of zero shear and loads are again applied, the sample quickly returns to the failure envelope.
The overall behavior can be explained if we consider the behavior at particle level. If there is no rotation of stresses during release, the particles would be forced to move in a plane (as in a uni-directional tests). This may allow them to become locked together and resist movement even as the stress is released. This locking limits densification, which in turn prohibits pore pressure generation. The grains remain locked until sufficient stress has been released to allow relative movement. By contrast, if a stress is applied laterally to the sample as the load is being released, it is difficult for the grains to remain locked together. When locking is prohibited, pore pressures will be developed as expected. Further investigation to quantify this behavior for model development is ongoing.

The Relationship between Shear Stress and Post-Triggering Strain Behavior

As shown in an earlier section, the excess pore pressure shows an reverse-linear related to the shear stress imposed at any point along a stress path. However, because the shear stress itself also acts as a driving stress, two conflicting effects are occurring at each point in a given stress path. The results of this research indicate that there is a range of shear stresses for which the balance between these two competing aspects result in high shear strain potential, as shown schematically in Figure 14.

Consider that for very small values of shear stress, the pore pressures are high and the sample is softened to a large extent, but the driving stresses that deform the sample are low. In the very high shear stress region, the driving stresses are high, but the pore pressure is reduced to a value below which softening can occur. In some region of moderate shear stresses, reasonable driving stresses are present and but pore pressures are still adequate for softening. This results in a range of SSR values with high strain potential.

![FIG 14: Relationship between pore pressure generation and driving shear stress](image-url)
Figure 15 is an example of a test series in which this optimal shear stress range high strain potential is evident. In this test series, it is not the sample with the highest CSR that shows the largest strains, but rather a sample experiencing more moderate shear stresses. While, this is one of two bi-directional linear tests that are particularly useful for demonstrating this phenomenon due to its simplicity, similar behavior is as apparent in the level-ground oval testing series as well.

This balance between pore pressure generation and driving stress has not been recognized widely in the past, if at all. In fact, these results seem at the surface to contradict other testing that has found that for soils loaded under level-ground one-directional conditions, an increase in loading level (CSR) always decreases the number of cycles required to achieve “failure”.

However, this is easily explained because in uni-directional level-ground testing the shear stresses necessarily range from zero to SSR\(_{\text{max}}\) twice in each cycle, thus there is little chance of recognizing or differentiating any region of higher shear strain potential. By contrast, in multi-directional testing it is possible to test a sample with a very only a very small range of shear stresses. For example, the test shown in Figures 4, 6, and 7 has a more limited stress range and a level-ground circular test has only a single shear stress magnitude.

**Pore Pressure and Liquefaction Triggering**

There are several tests presented in this paper that developed large strains while exhibiting \(r_{\text{u\text{lim}}}\) values far less than 1.0. The stress paths of these tests maintain a relatively large shear stress throughout each cycle, and thus cannot produce large pore pressures due to the reverse relationship between shear stress and pore pressure. Indeed, this behavior was common throughout the bi-directional testing series.
performed because many stress paths tested did not approach a point of zero shear stress.

The idea of \( r_u = 1.0 \) as a definition of liquefaction triggering results from the preponderance of uni-directional testing in liquefaction research. Because there is complete removal of shear strains (SSR=0) twice in each uni-directional test, \( r_{u,\text{lim}} \) values of approximately 1.0 occur regularly. However, results from this testing program indicate that once an \( r_u \) value of approximately 0.65 or higher is reached, sufficient softening occurs to start a positive feedback system—values near 1.0 are not required. The onset of softening allows increased strains, which in turn allows for more efficient densification and increasingly larger strains and softening in each cycle. It can be seen that the strains in Figures 8(E) begin to accelerate at \( r_u \) values closer to 0.7 than 1.0. In fact, the acceleration of strains at \( r_u \) values well below 1.0 has been commonly seen in uni-directional testing and has even been directly incorporated into models (Iai, 1992). In light of this information, it is important that models can account for softening and degradation directly and do not require pore pressure ratios approaching 1.0

**GETTING STARTED WITH THE DATA**

**The Bi-directional Linear Series**

Perhaps the most straightforward of all the data to work with is the bi-directional linear series. The data plots are easy to understand and look the most like the uni-directional tests that the researcher is most familiar with. As the \( \alpha \)-value in the perpendicular direction is increased, the following behaviors should be observed.

a. When a small \( \alpha \)-value is first applied, permanent strains should start accumulating in the downhill direction and the maximum pore pressure in the test should start to drop. The resistance to liquefaction should be dependent on the CSR and density as in the uni-directional testing.

b. As the \( \alpha \)-value increase, the liquefaction resistance is still reliant on a sufficient CSR value, but the strains start to increase with increasing \( \alpha \). At some point, as the \( \alpha \)-value continues to increase the strains start to slow and the pore pressures drop to the point where softening is inhibited. It may happen that at the point of optimal \( \alpha \)-value in terms of strain potential, the maximum pore pressure may appear lower than expected based on the failure envelope. This is consistent with the data and may be a result of dilation as the material strains quickly.

c. Ultimately the \( \alpha \)-value will be large enough that the maximum pore pressure ratio will not reach the value at which the sample can soften.

d. If the researcher wants to work with a specific data series, the series presented in this paper (50k, 55k, 61k) is recommended. This series is composed of medium density tests with increasing \( \alpha \)-value and a constant degree of rotation. The data is very consistent across all the bi-directional linear tests.

**The Level Ground Oval Series**

Another series useful for early work with the data set is the level ground oval series such as was studied in this testing program and by Ishihara and Yamazaki. Again the
data is relatively straightforward and important trends in the data are captured in this series. As the aperture ratio is increased through the addition of load in a second direction, the load paths will change from uni-directional testing, to oval shaped, and ultimately to circular paths. As this happens the following behaviors should be observed:

a. Moving from uni-directional testing to a very narrow oval should provide a marked change in behavior with the liquefaction resistance changing significantly
b. As the aperture ratio continues to increase the resistance should continue to drop, though the change in resistance should be reduced as the aperture ratio increases.
c. In some cases the strains with increased loading may remain remarkably constant. This is the case when the majority of the load path is moving through the range of high shear strain potential discussed above.
d. Ultimately, depending on the CSR values chosen, the shear stresses will be too high to allow pore pressure development and the triggering will stop.
e. It is of interest to also perform a series of circular tests with increasing load. When the CSR is very low, the sample should not trigger due to insufficient load. Ultimately the strains should increase, perhaps rapidly. At the point of maximum shear stress, the pore pressures may appear lower than would be expected. As the load continues to increase, the strains beyond the initial movement will start to diminish. Ultimately, the load will be too high for pore pressures to develop and triggering will again stop.

SUMMARY

While the behavior of liquefiable soils under multi-directional loading is far too complex to fully address here, the general behaviors discussed in this paper can be summarized.

- The addition of shear stress in a second horizontal direction tends to result in quicker attainment of $r_{u,\text{lim}}$, though the actual values of $r_{u,\text{lim}}$ may be lower. The biggest change in liquefaction resistance occurs when the load is first applied in the second direction. Further increases in the load in the second direction typically continues to lower triggering resistance, but with declining effect.
- As with uni-directional loading, increasing the magnitude of the shear stresses tends to reduce the number of cycles to the limiting pore pressure. However, there are notable exceptions
- After the limiting pore pressure ($r_{u,\text{lim}}$) has been achieved in a given test, the pore pressures measured at any point during a cycle show a (roughly) reverse linear relationship with the shear stresses at each point. This relationship can be estimated from the failure envelope, though some exceptions do apply.
- The assumption that the excess pore pressure within a sample must approach the total vertical stress (resulting in $r_u \approx 1.0$) in order for “liquefaction” to be achieve must be reassessed based on results showing that a large number of tests exhibited large strains with relatively low maximum pore pressures ($r_{u,\text{max}}=0.7$ or less). This logically follows once it is recognized that (a) there is a relationship
between minimum shear stress and maximum excess pore pressure and (b) few multi-directional tests experience points of zero shear stress.

- As with uni-directional loading, a sufficiently large initial static shear stress ratio (i.e. high $\alpha$) can increase resistance to liquefaction triggering by greatly reducing the degree of stress reversal and, as a result, the potential for excess pore pressure generation. However, a small amount of reversal can produce large pore pressures in some multi-directional load paths incorporating rotation. This can result in an abrupt change in behavior.
- The lag in pore pressure generation that occurs during the unloading phase of each cycle in uni-directional testing is reduced or eliminated if rotation of the shear stresses in the horizontal plane occurs during the shear stress release.

**ACKNOWLEDGMENTS**

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**REFERENCES**


**TABLE 1: Summary of Bi-Directional Testing Performed**

<table>
<thead>
<tr>
<th>Plan View Test Path (dashed line=α)</th>
<th>Medium Density (D_r&lt;70%)</th>
<th>Dense (D_r&gt;70%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low CSR (&lt;0.3)</td>
<td>High CSR (&gt;0.3)</td>
</tr>
<tr>
<td></td>
<td>Low CSR (&lt;0.3)</td>
<td>High CSR (&gt;0.3)</td>
</tr>
<tr>
<td>1-Directional Linear Paths</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extensive one-directional testing on Monterey #0/30 sand using the UCB Bi-directional device was performed as a companion project. For test data refer to Wu (2002)</td>
<td>Ms66cyck, 78, 0.51, 0.29, A</td>
<td>Ms67cyck, 83, 0.48, 0.15, A</td>
</tr>
<tr>
<td>2-Directional Linear Paths</td>
<td>Ms20cyck, 64, 0.24, 0.08, A</td>
<td>Ms50cyck, 59, 0.43, 0.24, A</td>
</tr>
<tr>
<td></td>
<td>Ms61cyck, 63, 0.26, 0.13, A</td>
<td>Ms55cyck, 65, 0.62, 0.35, A</td>
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<tr>
<td></td>
<td>Ms56cyck, 70, 0.46, 0.40, A</td>
<td></td>
</tr>
<tr>
<td>Figure-8 Paths</td>
<td>Ms33cyck, 56, 0.25, 0.02, A</td>
<td>Ms42cyck, 62, 0.39, 0.03, A</td>
</tr>
<tr>
<td></td>
<td>Ms40cyck, 68, 0.24, 0.02, A</td>
<td>Ms51cyck, 63, 0.44, 0.22, A</td>
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<tr>
<td></td>
<td>Ms41cyck, 63, 0.23, 0.01, A</td>
<td>Ms52cyck, 65, 0.50, 0.46, A</td>
</tr>
<tr>
<td></td>
<td>Ms48cyck</td>
<td>Ms48cyck</td>
</tr>
<tr>
<td></td>
<td>Ms38cyck, 63, 0.23, 0.08, A</td>
<td>Ms51cyck, 63, 0.44, 0.22, A</td>
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<tr>
<td></td>
<td>Ms41cyck, 63, 0.23, 0.01, A</td>
<td>Ms52cyck, 65, 0.50, 0.46, A</td>
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<tr>
<td></td>
<td>Ms33cyck</td>
<td>Ms48cyck</td>
</tr>
<tr>
<td></td>
<td>Ms33cyck</td>
<td>Ms37cyck, 61, 0.36, 0.18, A</td>
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<td>Ms19cyck, 78, 0.23, 0.12, A</td>
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<td>Ms26cyck, 80, 0.19, 0.20, A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ms65cyck, 83, 0.47, 0.07, A</td>
</tr>
</tbody>
</table>

Data shown for each test: name, relative density, cyclic stress ratio in maximum direction (CSR_{max}), normalized initial static shear Stress (α), Data quality class

Each test is first noted in the most appropriate category. Tests that are useful for assessing additional categories also are noted in italics in the appropriate category.
TABLE 1 (cont.): Summary of Bi-Directional Testing Performed

<table>
<thead>
<tr>
<th>Plan View Test Path (1)</th>
<th>Medium Density (D_r&lt;70%)</th>
<th>Dense (D_r&gt;70%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low CSR (&lt;0.3)</td>
<td>High CSR (&gt;0.3)</td>
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<td>Low CSR (&lt;0.3)</td>
<td>High CSR (&gt;0.3)</td>
</tr>
<tr>
<td></td>
<td>High CSR (&gt;0.3)</td>
<td></td>
</tr>
</tbody>
</table>

**Oval/Circular Paths**

- **Ms23cyck, 68**, 0.21, 0.02, A
- **Ms30cyck, 68**, 0.23, 0, A
- **Ms31cyck, 65**, 0.25, 0, A
- **Ms43cyck, 48**, 0.12, 0, A

- **Ms44cyck, 68**, 0.40, 0.02, A
- **Ms34cyck, 70**, 0.35, 0.02, A

- **Ms58cyck, 80**, 0.27, 0.05, B
  *Large drop in one vertical LVDT*

- **Ms68cyck, 82**, 0.44, 0.05, A

**Miscellaneous Paths (all medium density, low CSR)**

- **Ms32cyck, Dr =62%, B**
- **Ms27cyck, Dr =62%, A**
- **Ms48cyck, Dr =62%, A**

Data shown for each test: name, relative density, cyclic stress ratio in maximum direction (CSR_{max}), normalized initial static shear Stress (\(\alpha\)), Data quality class

Each test is first noted in the most appropriate category. Tests that are useful for assessing additional categories also are noted in italics in the appropriate category.
As multi-dimensional constitutive models are calibrated and compared with this data, it is recommended that these types of series are performed before those incorporating the figure-8 type loads for several reasons. First, the behavior is predictable and relatively well behaved. The only exception to this high degree of predictability is seen in tests with the optimal shear stress ratio (discussed below) which exhibit very large strains. In these cases the pore pressures are lower than would be expected as the grains remain in highly dilated state. Second, there is no effect of initial shear stress or loading directivity. Lastly, the stresses do not approach zero where small changes in the loading produce very large changes in behavior.

Figure 13: Uni-Directional simple shear test #ms15j
(D_r=54%, \( \sigma_v' \)=95kPa, CSR=0.20 (Wu et al., 2002)